

# Chiral quark model analysis of nucleon quark sea isospin asymmetry and spin polarization

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**Abstract.** We analyze recent measurements of the nucleon quark sea isospin asymmetry in terms of the chiral quark model. The new measurements indicate that the SU(3) model with modest symmetry breaking and no  $\eta'$  Goldstone boson gives a satisfactory description of data. We also discuss the matching parameter for the axial-vector current. Finally, we analyze the nucleon quark spin polarization measurements directly in the chiral quark model without using any SU(3) symmetry assumption on the hyperon axial-vector form factors. The new data indicate that the chiral quark model gives a remarkably good and consistent description of all low energy baryon measurements.

## 1 Introduction

The parameterization of low energy hadron structure by the chiral quark model ( $\chi$ QM), suggested by Manohar and Georgi [1] have enjoyed rising interest recently [2–15]. Especially, the emission and absorption of pseudoscalar Goldstone bosons (GBs) from the quarks lead to a spin depolarization that seems consistent with measurements of the quark spin polarization in nucleons, axial-vector form factors and magnetic moments.

The  $\chi$ QM Lagrangian for the quark-GB interaction can be written, to lowest order, as

$$\mathcal{L} = g_8 \bar{\mathbf{q}} \Phi \gamma^5 \mathbf{q}, \quad (1)$$

where  $g_8$  is a coupling constant,

$$\mathbf{q} = \begin{pmatrix} u \\ d \\ s \end{pmatrix}, \text{ and } \Phi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \beta \frac{\eta}{\sqrt{6}} & \pi^+ & \alpha K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \beta \frac{\eta}{\sqrt{6}} & \alpha K^0 \\ \alpha K^- & \alpha \bar{K}^0 & -\beta \frac{2\eta}{\sqrt{6}} \end{pmatrix}$$

The GBs of the  $\chi$ QM are here denoted by the  $0^-$  meson names  $\pi, K, \eta, \eta'$ , as is usually done. We have introduced two SU(3) symmetry breaking parameters,  $\alpha$  and  $\beta$ , which allow for different strengths of production of GBs containing strange quarks.

To account for the quark sea isospin asymmetry as measured by the NA51 Collaboration [16] and the New Muon Collaboration (NMC) [17,18], Cheng and Li [3] suggested the introduction of a broken U(3) symmetric model with nine GBs. The ninth GB, the  $\eta'$ , should couple with a relative strength  $\zeta$ , that is different from the strength of

the octet GBs (when  $\alpha = 1$  and  $\beta = 1$ ). The interaction Lagrangian has the form  $\mathcal{L}' = g_8 \zeta \frac{1}{\sqrt{3}} \bar{\mathbf{q}} \eta' \gamma^5 \mathbf{q}$ .

The probability of transforming a quark with spin up by one interaction can then be expressed by the functions

$$|\psi(u^\uparrow)|^2 = \frac{1}{6} a(3 + \beta^2 + 2\zeta^2) \hat{u}^\downarrow + a \hat{d}^\downarrow + a \alpha^2 \hat{s}^\downarrow, \quad (2)$$

$$|\psi(d^\uparrow)|^2 = a \hat{u}^\downarrow + \frac{1}{6} a(3 + \beta^2 + 2\zeta^2) \hat{d}^\downarrow + a \alpha^2 \hat{s}^\downarrow, \quad (3)$$

$$|\psi(s^\uparrow)|^2 = a \alpha^2 \hat{u}^\downarrow + a \alpha^2 \hat{d}^\downarrow + \frac{1}{3} a(2\beta^2 + \zeta^2) \hat{s}^\downarrow. \quad (4)$$

The coefficient of a quark  $\hat{q}^\downarrow$  is the transition probability to  $q^\downarrow$ . The parameter  $a$  ( $a \propto g_8^2$ ) measures the probability of emission of a GB from a quark and is sometimes called the fluctuation (or probability) parameter.

The NA51 experiment gave for the quark sea isospin asymmetry the value  $\bar{u}/\bar{d} = 0.51 \pm 0.09$  at  $x = 0.18$  [16]. This excludes that the value  $\zeta = 0$  can be used, at least for  $\beta \simeq 1$ .

However, recently there has been a remeasurement of both the  $\bar{u}/\bar{d}$  asymmetry as well as of the  $\bar{u} - \bar{d}$  asymmetry by the NuSea Collaboration<sup>1</sup> [19,20]. Their data differ substantially from the earlier measurements. It therefore seems appropriate to re-evaluate the range of the parameters of the  $\chi$ QM in view of the new data. In this paper, we undertake such a re-evaluation in the framework of the broken SU(3) symmetric  $\chi$ QM, and confront the model with the spin polarization measurements of the nucleons.

The new measurements lead to the conclusion that there is no need for a ninth GB. Thus we can set  $\zeta = 0$ . The magnetic moment data show that there is no need to distinguish the  $\eta$  GB from the  $\pi$  GBs. Thus only two parameters,  $a$  and  $\alpha$ , remain in the model. We find the best fit for those parameters, and show that these values are in agreement with the matching parameter  $g_a = 1$ ,

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in accordance with the analysis of Weinberg [21, 22] and with the nucleon axial-vector form factor  $g_A^{np}$ . Since the model parameters can be determined from nucleon data alone, we can study the nucleon quark spin polarization measurements in this model without introducing any assumption on the axial-vector form factor  $a_8$ . This allows in principle the possibility to study the gluon polarization in the nucleon.

The outline of our paper is as follows. In Sect. 2 we discuss the quark sea isospin asymmetry and also the so called matching parameter  $g_a$  introduced by Manohar and Georgi [1]. This parameter is closely related to the integral expression for the parameter  $a$  [2]. In Sect. 3 we investigate the parameter space for the  $\chi$ QM using the neutron-proton axial-vector form factor  $g_A^{np}$  and confront the model with the best measurements for the quark spin structure functions. This eventually leads to a determination of the nucleon gluon spin polarization. Finally, in Sect. 4, we present a summary of our analysis and also the main conclusions.

## 2 Nucleon quark sea isospin asymmetry

As mentioned above, the quark sea isospin asymmetry in the nucleon has previously been measured by the NA51 experiment to be  $\bar{u}/\bar{d} = 0.51 \pm 0.09$  at  $x = 0.18$  [16]. The new measurement by the NuSea Collaboration [19,20] gives this asymmetry for a range of  $x$ -values. Their value for the  $\bar{u}/\bar{d}$  asymmetry at  $Q = 7.35$  GeV can be accurately fitted by the formula

$$\bar{d}(x)/\bar{u}(x) = 1 + 1120x^{2.75}(1-x)^{15} \quad (5)$$

in the region  $0.02 < x < 0.345$  (see Fig. 1). Lacking independent measurements of  $\bar{u}(x)$  and  $\bar{d}(x)$ , we will here define the  $\bar{u}/\bar{d}$  asymmetry as

$$\bar{u}/\bar{d} \equiv \left( \int_0^1 \bar{d}(x)/\bar{u}(x) dx \right)^{-1}. \quad (6)$$

By integrating (5) over the given region, we obtain the asymmetry as

$$\bar{d}/\bar{u} \simeq \frac{\int_{0.02}^{0.345} \bar{d}(x)/\bar{u}(x) dx}{0.345 - 0.02} \approx 1.325,$$

whence  $\bar{u}/\bar{d} \approx 0.755^2$ .

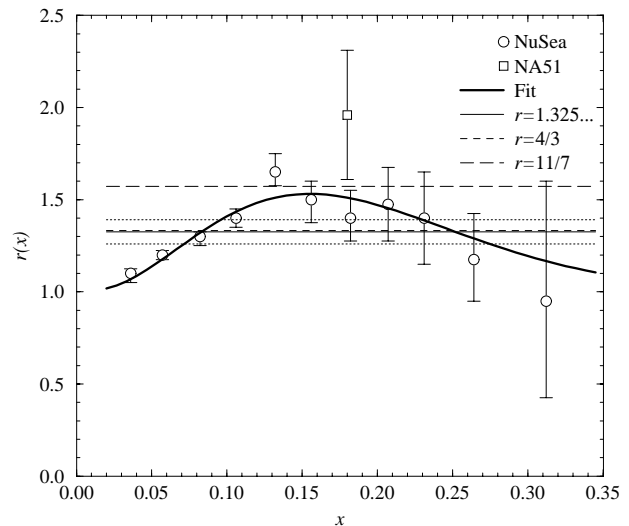
In the  $\chi$ QM, this asymmetry is given by [8]

$$\bar{u}/\bar{d} = \frac{21 + 2\xi + \xi^2}{33 - 2\xi + \xi^2}, \quad (7)$$

where  $\xi \equiv 2\zeta + \beta$ . Comparing this with the new result above shows that the asymmetry is compatible with the parameter  $\xi = 1$  for which

$$\bar{u}/\bar{d} = \frac{3}{4} = 0.75.$$

<sup>2</sup> Another possible definition of the  $\bar{u}/\bar{d}$  asymmetry is  $\bar{u}/\bar{d} \equiv \int_0^1 (\bar{d}(x)/\bar{u}(x))^{-1} dx$ . This definition leads to  $\bar{u}/\bar{d} \approx 0.765$ , which is almost the same as  $\bar{u}/\bar{d} \approx 0.755$

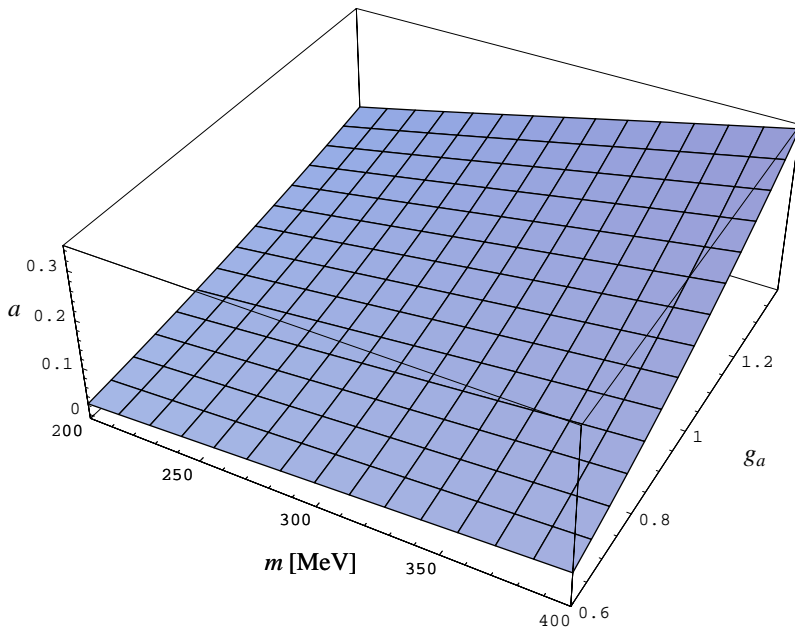


**Fig. 1.** The  $\bar{d}/\bar{u}$  ratio at  $Q = 7.35$  GeV.  $r(x) \equiv \bar{d}(x)/\bar{u}(x)$ , where  $0.02 < x < 0.345$ . The 11 data points marked with  $\circ$  were obtained by the NuSea Collaboration [19,20] and the data point marked with  $\square$  was obtained by the NA51 Collaboration [16]. The thick solid curve was obtained by the NuSea Collaboration by fitting to the 11 experimental data points (not including the data point from the NA51 experiment). The analytical expression for the fitted curve is  $1 + 1120x^{2.75}(1-x)^{15}$ . The thin solid line is the average  $\bar{d}/\bar{u}$  ratio, which is about 1.325, and the dotted lines are the corresponding 5 % errors. The dashed line shows the  $\bar{d}/\bar{u}$  ratio for the SU(3)  $\chi$ QM and the long dashed line shows the  $\bar{d}/\bar{u}$  ratio for the SU(2)  $\chi$ QM (*i.e.* the  $\chi$ QM with just pions as GBs)

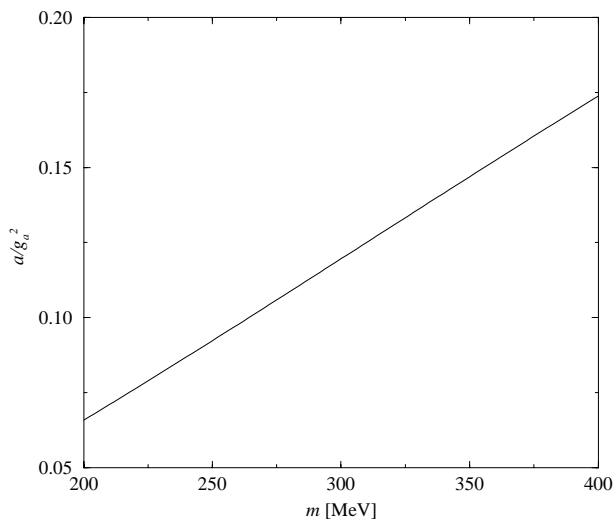
The value  $\xi = 2\zeta + \beta = 1$  implies that for a value of  $\beta$  around 1, we can put  $\zeta = 0$ , corresponding to complete  $\eta'$  suppression. Evidence for a strong  $\eta'$  suppression has also been suggested by Song [9] and is discussed in [5, 7, 10]. The value  $\zeta = 0$  is welcome, since our understanding from QCD concerning the role of the  $\eta'$  is that it is not a GB, but related to instantons via the axial anomaly [23].

In Fig. 1 we see that the  $\bar{d}/\bar{u}$  asymmetry for the SU(3)  $\chi$ QM coincides remarkably well with the experimental result. In the paper by the NuSea Collaboration [20], it is indicated in their Fig. 3, and commented on in the text, that the  $\chi$ QM is not compatible with data. However, their quoted value of 11/7 for the  $\bar{d}/\bar{u}$  asymmetry in the  $\chi$ QM is relevant only for the SU(2)  $\chi$ QM (corresponding to  $\beta = 0$  and  $\zeta = 0$ , *i.e.*  $\xi = 0$  in (7)), and not for the SU(3)  $\chi$ QM. We should thus instead interpret the new result as an experimental verification of the presence of  $\eta$  GBs in the  $\chi$ QM.

We next study the quark sea isospin asymmetry  $\bar{u} - \bar{d}$  appearing in the Gottfried sum-rule [24]. Here the previous value is  $\bar{u} - \bar{d} = -0.15 \pm 0.04$ , which was obtained by the NMC [17, 18]. In [20], the NuSea Collaboration has measured this to be  $\bar{u} - \bar{d} = -0.100 \pm 0.024$  at  $Q = 7.35$  GeV. This is 2/3 of the value deduced by the NMC. We can now use this new measurement to estimate the parameter  $a$  in the  $\chi$ QM. We will use the parameter values  $\beta = 1$  and  $\zeta = 0$ , *i.e.*  $\xi = 1$ . From the formula of the



**Fig. 2.** The probability parameter  $a$  plotted against the quark mass  $m$  and the matching parameter (the quark axial-vector current coupling constant)  $g_a$



**Fig. 3.** The ratio  $a/g_a^2$  as a function of the quark mass  $m$ .  $a/g_a^2 = f(m)$ , where  $f(m)$  is in integral form. In the indicated region of  $m$ , the function  $f(m)$  can be fitted with the linear function  $c_0 + c_1 m$ , where  $c_0 \approx -0.043$  and  $c_1 \approx 0.00054 \text{ MeV}^{-1}$

$\bar{u} - \bar{d}$  asymmetry in the  $\chi$ QM [8]

$$\bar{u} - \bar{d} = a \left( \frac{2\zeta + \beta}{3} - 1 \right) = a \left( \frac{\xi}{3} - 1 \right), \quad (8)$$

we then obtain  $a = 0.150 \pm 0.036$ .

In the  $\chi$ QM,  $\bar{u}/\bar{d}$  and  $\bar{u} - \bar{d}$  are both independent of  $\alpha$ . The parameters  $\beta$  and  $\zeta$  determine the relative mixing of the isospin triplet  $\bar{\pi}$  and the isospin singlets  $\eta$  and  $\eta'$ .

The probability parameter  $a$  has also been calculated using the chiral field theory approach [2]. The result is

$$a = \frac{g_8^2}{32\pi^2} \int_0^1 \theta(\Lambda_{\chi\text{SB}}^2 - \tau(z)) z \times \left\{ \ln \frac{\Lambda_{\chi\text{SB}}^2 + m_\pi^2}{\tau(z) + m_\pi^2} + m_\pi^2 \left[ \frac{1}{\Lambda_{\chi\text{SB}}^2 + m_\pi^2} - \frac{1}{\tau(z) + m_\pi^2} \right] \right\} \times dz, \quad (9)$$

where  $g_8 \equiv 2mg_a/f_\pi$ ,  $\tau(z) \equiv m^2 z^2/(1-z)$ , and  $\theta$  is the Heaviside function. The parameters  $m$ ,  $m_\pi$ ,  $f_\pi$ , and  $\Lambda_{\chi\text{SB}}$  are the quark mass, the GB mass, the pseudoscalar decay constant, and the chiral symmetry breaking scale, respectively. In the integral expression above for the parameter  $a$ , the so called ‘‘matching parameter’’,  $g_a$ , introduced by Manohar and Georgi [1], occurs. The parameter  $g_a$  is sometimes also called the quark axial-vector current coupling constant. Using the value  $a = 0.15$  and the parameters  $m = 363 \text{ MeV}$ ,  $m_\pi = 140 \text{ MeV}$ ,  $f_\pi = 93 \text{ MeV}$ ,  $\Lambda_{\chi\text{SB}} \simeq 4\pi f_\pi = 1169 \text{ MeV}$ , one obtains  $g_a \approx 0.987 \sim 1$  from solving (9). This is in good agreement with the arguments of Weinberg [21, 22], that to lowest order in  $1/N_c$ , where  $N_c$  is the number of colors, the value of  $g_a$  should be 1.

It should be noted that the matching parameter  $g_a$  and the value of the parameter  $a$  are closely related. In Fig. 2 we have displayed  $a$  as a function  $m$  and  $g_a$ . This is further exemplified in Fig. 3, where we show how  $a/g_a^2$  varies with  $m$ . We observe that in the relevant region  $a$  scales as  $g_a^2$  for a fixed value of  $m$ . This dependence is neglected by several authors, who calculate the value of the axial-vector coupling constant of the nucleon from the difference of the quark spin polarizations  $\Delta q$  as  $g_A^{np} = \Delta u - \Delta d$ . However, when the matching parameter  $g_a \neq 1$  the expression for  $g_A^{np}$  must be multiplied by  $g_a$ , *i.e.*  $g_A^{np} = g_a (\Delta u - \Delta d)$  [1, 13].

**Table 1.** The quark spin polarizations and  $a_i$ , where  $i = 0, 3, 8$ .  $a_0 = \Delta\Sigma \equiv \Delta u + \Delta d + \Delta s$ ,  $a_3 = g_A^{np} \equiv \Delta u - \Delta d$ , and  $a_8 \equiv \Delta u + \Delta d - 2\Delta s$ . The experimental values from [27] have been obtained assuming  $g_A^{np} = 1.2573 \pm 0.0028$  and  $a_8 = 0.601 \pm 0.038$  and the experimental values from [34] have been obtained assuming  $g_A^{np} = 1.2573 \pm 0.0028$  and  $F/D = 0.575 \pm 0.016$ . The data for the  $\chi$ QM are calculated using  $a = 0.15$

Quantity	Experimental value	NQM	$\chi$ QM $\alpha \approx 0.54$	$\chi$ QM $\alpha = \frac{2}{3}$
$\Delta u$	$0.83 \pm 0.03$ [27]	$\frac{4}{3}$	0.86	0.83
$\Delta d$	$-0.43 \pm 0.03$ [27]	$-\frac{1}{3}$	-0.40	-0.39
$\Delta s$	$-0.10 \pm 0.03$ [27] $-0.09 \pm 0.02$ [34]	0	-0.04	-0.07
$a_0$	$0.31 \pm 0.07$ [27] $0.30 \pm 0.06$ [34]	1	0.41	0.37
$a_3$	$1.2601 \pm 0.0025$ [25]	$\frac{5}{3}$	1.2601 (input)	1.22
$a_8$	$0.601 \pm 0.038$ [36]	1	0.54	0.57

Consider next the quark spin polarizations. In the  $\chi$ QM, they are given by [7, 8]

$$\Delta u = \frac{4}{3} - a \left( \frac{7}{3} + \frac{4}{3}\alpha^2 + \frac{4}{9}\xi' \right), \quad (10)$$

$$\Delta d = -\frac{1}{3} - a \left( \frac{2}{3} - \frac{1}{3}\alpha^2 - \frac{1}{9}\xi' \right), \quad (11)$$

$$\Delta s = -a\alpha^2, \quad (12)$$

where  $\xi' \equiv \beta^2 + 2\zeta^2$ . Thus for  $\beta = 1$  and  $\zeta = 0$ , we have  $\xi' = 1$ .

Using the quark spin polarizations, the observable  $g_A^{np}$ , expressed in terms of the  $\chi$ QM, becomes

$$g_A^{np} = g_a (\Delta u - \Delta d) = \frac{5}{3}g_a \left[ 1 - a \left( 1 + \alpha^2 + \frac{1}{3}\xi' \right) \right]. \quad (13)$$

From our previous discussion, we have seen that the parameters are compatible with  $g_a = 1$ , which will be used from now on. We will also fix the parameter  $\beta$  to 1, which seems to be favored by the magnetic moment data [8, 10], and hence the parameter  $\zeta$  to 0, which implies that  $\xi' = 1$ . We have then only one free parameter  $\alpha$  to fit to  $g_A^{np} = 1.2601 \pm 0.0025$  [25]. The result is  $\alpha \approx 0.54$ . This can, of course, not be taken as compulsory, since the model would not allow  $g_A^{np}$  to be determined that well. From magnetic moment data slightly higher values of  $\alpha$ , up to 0.7, are favored [8, 10]. Since  $\alpha$  is the suppression factor for kaon GB emission, it can be argued that  $\alpha$  is of the order  $m/m_s \simeq 2/3$  [4, 5, 7].

In Table 1 we list the quark spin polarizations calculated from (10)–(12) with  $\xi' = 1$  and  $\alpha \approx 0.54$  and  $\alpha = 2/3$ , respectively, as well as some other related quantities.

### 3 Nucleon quark spin polarization

The nucleon quark spin polarizations are usually analyzed by the spin dependent quark structure functions  $g_1^p$  and  $g_1^n$ . In these analyses, the values of  $g_A^{np}$  and  $a_8$  are commonly used [26, 27]. The experimental value for  $a_8$  is obtained from hyperon semileptonic decays using the assumption of SU(3) flavor symmetry. However, when the

SU(3) symmetry is broken it is not clear how to connect the axial-vector form factors to the quark spin polarizations. Since the parameters in the  $\chi$ QM can be fixed without using the hyperon axial-vector form factors, it offers an independent way to analyze the nucleon spin. We thus express the nucleon quark spin polarization directly in terms of the  $\chi$ QM quark spin polarizations, avoiding the use of any SU(3) symmetry assumptions for the value of  $a_8$ . For our analysis, we will use the formulas from the analysis of Ellis and Karliner [26, 27].

Define the integrals

$$\Gamma_1^p(Q^2) \equiv \int_0^1 g_1^p(x, Q^2) dx, \quad (14)$$

$$\Gamma_1^n(Q^2) \equiv \int_0^1 g_1^n(x, Q^2) dx, \quad (15)$$

which are the first moments of the proton and neutron spin structure functions, respectively. These integrals can be expressed in terms of the quark spin polarizations, using the evolution equations for arbitrary  $Q^2$ , by means of two functions,  $f = f(Q^2)$  and  $h = h(Q^2)$ , in the form

$$\Gamma^p(Q^2) = \frac{1}{9}\Delta u(Q^2)(f+h) + \frac{1}{18}\Delta d(Q^2)(2h-f) + \frac{1}{18}\Delta s(Q^2)(2h-f), \quad (16)$$

$$\Gamma^n(Q^2) = \frac{1}{9}\Delta d(Q^2)(f+h) + \frac{1}{18}\Delta u(Q^2)(2h-f) + \frac{1}{18}\Delta s(Q^2)(2h-f). \quad (17)$$

The functions  $f$  and  $h$  depend on the number of flavors and the renormalization scheme. In the  $\overline{\text{MS}}$  scheme for  $N_f = 3$  flavors, the functions  $f$  and  $h$  are given by [28]

$$f(\alpha_s(Q^2)) = 1 - \frac{\alpha_s(Q^2)}{\pi} - 3.5833 \left( \frac{\alpha_s(Q^2)}{\pi} \right)^2 - 20.2153 \left( \frac{\alpha_s(Q^2)}{\pi} \right)^3 - \mathcal{O}(130) \left( \frac{\alpha_s(Q^2)}{\pi} \right)^4 + \dots \quad (18)$$

and [29]

$$h(\alpha_s(Q^2)) = 1 - \frac{\alpha_s(Q^2)}{\pi} - 1.0959 \left( \frac{\alpha_s(Q^2)}{\pi} \right)^2 - 3.7 \left( \frac{\alpha_s(Q^2)}{\pi} \right)^3 + \dots \quad (19)$$

Ellis and Karliner [26, 27] use the combinations  $g_A^{np} \equiv \Delta u - \Delta d = 1.2573 \pm 0.0028$  and  $a_8 \equiv \Delta u + \Delta d - 2\Delta s = 0.601 \pm 0.038$  to evaluate  $\Delta\Sigma(Q^2) \equiv \Delta u(Q^2) + \Delta d(Q^2) + \Delta s(Q^2)$ . Their most recent value is  $\Delta\Sigma(Q^2) = 0.31 \pm 0.07$  at a renormalization scale of  $Q^2 = 10 \text{ GeV}^2$  [27].

However, as mentioned above, rather than using the experimental values for the axial-vector form factors to evaluate  $\Delta\Sigma(Q^2)$ , we would like to analyze the spin structure integrals directly in terms of the  $\chi$ QM.

**Table 2.** The total quark spin polarization  $\Delta\Sigma(Q^2)$  at  $Q^2 = 5 \text{ GeV}^2$ . The data for the  $\chi\text{QM}$  are calculated using  $\alpha_s(Q^2 = 5 \text{ GeV}^2) = 0.287 \pm 0.020$  (corresponding to  $\alpha_s(Q^2 = M_Z^2) = 0.118 \pm 0.003$  [25])

		Experiments		
		Experimental value	$\chi\text{QM}$ $\alpha \approx 0.54$	$\chi\text{QM}$ $\alpha = \frac{2}{3}$
$\Gamma_1^p$	SMC	$0.132 \pm 0.017$	$0.295 \pm 0.171$	$0.317 \pm 0.171$
	World average	$0.142 \pm 0.011$	$0.395 \pm 0.110$	$0.417 \pm 0.110$
$\Gamma_1^n$	SMC	$-0.048 \pm 0.022$	$0.289 \pm 0.221$	$0.257 \pm 0.221$
	World average	$-0.061 \pm 0.016$	$0.156 \pm 0.161$	$0.126 \pm 0.161$

Expressed in terms of the  $\chi\text{QM}$  parameters, we obtain the following relations

$$\begin{aligned} \Gamma_1^p(Q^2) &= \frac{1}{12}f(g_A^{np} + \frac{1}{3}a_8) + \frac{1}{9}h\Delta\Sigma(Q^2) \\ &= \frac{1}{6}f[1 - a(\frac{4}{3} + \frac{2}{3}\alpha^2 + \frac{1}{3}\xi')] \\ &\quad + \frac{1}{9}h\Delta\Sigma(Q^2), \end{aligned} \quad (20)$$

$$\begin{aligned} \Gamma_1^n(Q^2) &= \frac{1}{12}f(-g_A^{np} + \frac{1}{3}a_8) + \frac{1}{9}h\Delta\Sigma(Q^2) \\ &= -\frac{1}{9}f[1 - a(\frac{1}{2} + \frac{2}{3}\alpha^2 + \frac{1}{3}\xi')] \\ &\quad + \frac{1}{9}h\Delta\Sigma(Q^2). \end{aligned} \quad (21)$$

This is consistent with the fact that in all renormalization schemes the combinations  $g_A^{np} \equiv \Delta u - \Delta d$  and  $a_8 \equiv \Delta u + \Delta d - 2\Delta s$  are independent of  $Q^2$ . Above they appear in the combinations  $\pm g_A^{np} + \frac{1}{3}a_8$ , which are likewise independent of  $Q^2$ .

Using the parameter values found in the previous analyses, we can now extract the value for  $\Delta\Sigma(Q^2)$  from data of  $\Gamma_1^p(Q^2)$  and  $\Gamma_1^n(Q^2)$ . We use the published world average values and the Spin Muon Collaboration (SMC) data [30] for this analysis, which is performed at  $Q^2 = 5 \text{ GeV}^2$ . The results are found in Table 2. From this table, we can see that the world average values do not give consistent values for  $\Delta\Sigma(Q^2)$  from  $\Gamma_1^p(Q^2)$  and  $\Gamma_1^n(Q^2)$ , in contrast to the SMC measurements. In what follows, we therefore adopt the SMC measurements, that give for  $\Delta\Sigma(Q^2)$  the values  $0.292 \pm 0.139$  for  $\alpha \approx 0.54$  and  $0.287 \pm 0.139$  for  $\alpha = 2/3$ . Since these values overlap, we will use the mean value of them, *i.e.*

$$\Delta\Sigma(Q^2) = 0.29 \pm 0.14 \quad \text{at } Q^2 = 5 \text{ GeV}^2.$$

The total spin of the nucleon in QCD can be decomposed as [31]

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta L_q + \Delta g + \Delta L_g, \quad (22)$$

where  $\Delta\Sigma$  is the spin polarization contribution of the quarks,  $\Delta L_q$  is the orbital angular momentum of the quarks,  $\Delta g$  is the gluon contribution coming from the axial anomaly, and  $\Delta L_g$  is the orbital angular momentum of the gluons.

In the  $\chi\text{QM}$ , the orbital angular momentum of the quarks is compensated exactly by the spin polarization contribution of the sea quarks. Thus, if we write  $\Delta\Sigma = \Delta\Sigma_{\text{valence}} + \Delta\Sigma_{\text{sea}}$ , we have  $\frac{1}{2}\Delta\Sigma_{\text{sea}} + \Delta L_q = 0$  [11] and  $\Delta\Sigma_{\text{valence}} = 1$ . In the  $\chi\text{QM}$ , we can therefore interpret the spin of the nucleon as coming from the valence quarks. The gluonic degrees of freedom are accounted for essentially by the GBs below  $\Lambda_{\chi\text{SB}}$  and  $\Delta g = \Delta L_g = 0$ .

For higher  $Q$  values, the  $\chi\text{QM}$  breaks down and we should use ordinary QCD. We will therefore assume that the QCD calculation joins smoothly to the  $\chi\text{QM}$  and interpret the  $\Delta\Sigma \equiv \Delta u + \Delta d + \Delta s$  calculated in the  $\chi\text{QM}$  as the value of  $\Delta\Sigma(Q^2)$  for  $Q^2 \leq \Lambda_{\chi\text{SB}}^2$ . As  $Q^2$  increases, this value evolves according to the Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP) evolution equations [32]. In the Adler–Bardeen renormalization scheme [33], the quark spin polarizations can be written as

$$\Delta q(Q^2) = \Delta q - \frac{\alpha_s(Q^2)}{2\pi}\Delta g(Q^2), \quad (23)$$

for  $q = u, d, s$ . This leads to

$$\begin{aligned} \Delta\Sigma(Q^2) &= \Delta\Sigma - N_f \frac{\alpha_s(Q^2)}{2\pi}\Delta g(Q^2) \\ &= [1 - a(3 + 2\alpha^2 + \frac{1}{3}\xi')] \\ &\quad - N_f \frac{\alpha_s(Q^2)}{2\pi}\Delta g(Q^2), \end{aligned} \quad (24)$$

where  $N_f$  is the number of flavors. Here  $N_f = 3$ . The first term  $\Delta\Sigma$  is interpreted here to be given by the value in the  $\chi\text{QM}$ . From this it will then be possible to extract  $\Delta g$ .

With our parameterization, we can calculate  $\Delta\Sigma$  from the  $\chi\text{QM}$  to be  $\Delta\Sigma \approx 0.39$  as the mean value for the two different  $\alpha$ -values. We can then obtain an estimate of  $\Delta g$  from the value of  $\Delta\Sigma(Q^2) = 0.29 \pm 0.14$  at  $Q^2 = 5 \text{ GeV}^2$  obtained above. Using  $\alpha_s(Q^2 = 5 \text{ GeV}^2) = 0.287 \pm 0.020$ , which corresponds to  $\alpha_s(Q^2 = M_Z^2) = 0.118 \pm 0.003$  [25], this gives

$$\Delta g \approx 0.7 \pm 1.0 \quad \text{at } Q^2 = 5 \text{ GeV}^2.$$

This can be compared to the value found by the SMC, which is  $\Delta g = 1.7 \pm 1.1$  at  $Q^2 = 5 \text{ GeV}^2$  [30].

Our analysis of  $\Delta g$  does not depend on any assumption of SU(3) symmetry for the hyperon semileptonic decays. Nevertheless, the value  $a_8 \approx 0.57$  for  $\alpha = 2/3$  is in agreement with the value of  $F/D = 0.575 \pm 0.016^3$  used in the analysis of [34]. When the value of  $a_8$  changes with  $\delta a_8$ , the change in  $\Delta\Sigma(Q^2)$  is about  $-\frac{1}{4}\delta a_8$ , which explains the variation in Table 1.

## 4 Summary and conclusions

In light of the new measurements for the quark sea isospin asymmetry by the NuSea Collaboration, the broken SU(3)  $\chi$ QM gives an excellent fit to the data with the fluctuation parameter  $a = 0.15$  and the SU(3) breaking parameters  $\alpha \sim 0.6$  and  $\beta = 1$ , and the U(3) breaking parameter  $\zeta = 0$ , corresponding to no  $\eta'$  GB.

With these parameter values, the  $\chi$ QM accounts in a very satisfactory way for the quark sea isospin asymmetry, the nucleon quark spin polarizations, the axial-vector form factor of the nucleon, the magnetic moments of the nucleons and hyperons plus many other features of low energy hadron physics, such as the nucleon deep inelastic scattering structure functions. In addition, the new data are fully consistent with the matching parameter  $g_a = 1$ , as suggested by the analysis of Weinberg [21, 22]. (However, see also [35] for an extended discussion of possible values for  $g_a$ .)

The  $\chi$ QM also offers an independent way of analyzing the nucleon spin problem, relying only on nucleon data. This leads eventually to an estimate of  $\Delta\Sigma(Q^2) = 0.29 \pm 0.14$  at  $Q^2 = 5 \text{ GeV}^2$  and  $\Delta g \approx 0.7 \pm 1.0$  for the gluon spin polarization in the nucleon.

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<sup>3</sup>  $F/D = \frac{g_A^{np} + a_8}{3g_A^{np} - a_8}$